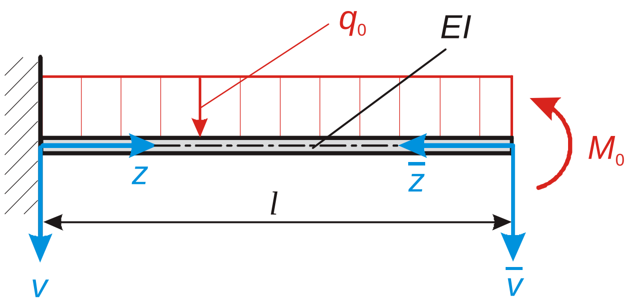
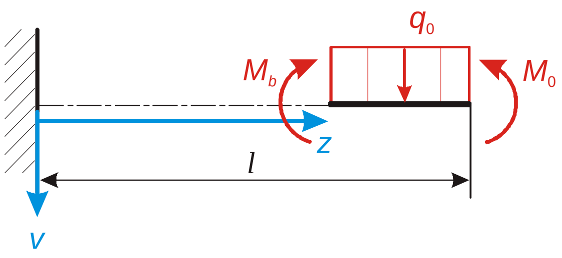
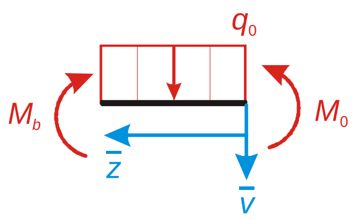
Gegenüberstellung der Lösungswege mit unterschiedlichen Koordinatensystemen bei der Ermittlung der Biegeverformung nach der DGL 2. Ordnung

<table>
<tr><th colspan="2">Beispiel</th></tr>
<tr><td>
</td><td>Geg.: q_0, M_0, l, EI = konst.

Ges.: In beiden Koordinatensystemen: Verläufe der Balkendurchbiegung und -neigung</td></tr>
<tr><th>Koordinatensystem v, z</th><th>Koordinatensytem $\bar{v}$, $\bar{z}$</th></tr>
<tr><td>
</td><td>
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$$EIv'' = -M_b(z) = -M_0 + \frac{q_0}{2}\left(l-z\right)^2 = -M_0 + q_0\left(\frac{l^2}{2} - l\,z + \frac{z^2}{2}\right)$$

$$EIv' \qquad = -M_0\,z + q_0\left(\frac{l^2}{2}z - \frac{l}{2}z^2 + \frac{1}{6}z^3\right) + C_1$$

$$EIv \qquad = -\frac{M_0}{2}z^2 + q_0\left(\frac{l^2}{4}z^2 - \frac{l}{6}z^3 + \frac{1}{24}z^4\right) + C_1\,z + C_2$$

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$$EI\bar{v}'' = -M_b(\bar{z}) = -M_0 + \frac{q_0}{2}\bar{z}^2$$

$$EI\bar{v}' \qquad = -M_0\,\bar{z} + \frac{q_0}{6}\bar{z}^3 + K_1$$

$$EI\bar{v} \qquad = -\frac{M_0}{2}\bar{z}^2 + \frac{q_0}{24}\bar{z}^4 + K_1\,\bar{z} + K_2$$

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<tr><td colspan="2">Randbedingungen, Integrationskonstanten</td></tr>
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$v\left(z=0\right)=0 \qquad C_2 = 0$

$v'\left(z=0\right)=0 \qquad C_1 = 0$

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$\bar{v}\left(\bar{z}=l\right)=0 \qquad -\frac{M_0\,l^2}{2} + \frac{q_0\,l^4}{24} + K_1\,l + K_2 = 0$

$\bar{v}'\left(\bar{z}=l\right)=0 \qquad -M_0\,l + \frac{q_0\,l^3}{6} + K_1 = 0$

$$K_1 = M_0\,l - \frac{q_0\,l^3}{6}$$

$$K_2 = -\frac{M_0\,l^2}{2} + \frac{q_0\,l^4}{8}$$

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<tr><td colspan="2">Lösung</td></tr>
<tr><td>

$$v'\left(z\right) = \frac{1}{EI}\left[-M_0\,z + \frac{q_0}{6}\left(z^3 - 3\,l\,z^2 + 3\,l^2\,z\right)\right]$$

$$v\left(z\right) = \frac{1}{EI}\left[-\frac{M_0}{2}z^2 + \frac{q_0}{24}\left(z^4 - 4\,l\,z^3 + 6\,l^2\,z^2\right)\right]$$

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$$\bar{v}'\left(\bar{z}\right) = \frac{1}{EI}\left[M_0\left(l-\bar{z}\right) - \frac{q_0}{6}\left(l^3 - \bar{z}^3\right)\right]$$

$$\bar{v}\left(\bar{z}\right) = \frac{1}{EI}\left[-\frac{M_0}{2}\left(l-\bar{z}\right)^2 + \frac{q_0}{24}\left(\bar{z}^4 - 4\,l^3\,\bar{z} + 3\,l^4\right)\right]$$

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Spezielle Werte	
$v'(z=l)=\frac{1}{EI}\left(-M_0\,l+\frac{q_0}{6}\,l^3\right)$ $v(z=l)\;=\frac{1}{EI}\left(-\frac{M_0}{2}\,l^2+\frac{q_0}{8}\,l^4\right)$	$\bar{v}'(\bar{z}=0)=\frac{1}{EI}\left(M_0\,l-\frac{q_0}{6}\,l^3\right)$ $\bar{v}(\bar{z}=0)\;=\frac{1}{EI}\left(-\frac{M_0}{2}\,l^2+\frac{q_0}{8}\,l^4\right)$
$v'(z=l)=-\bar{v}'(\bar{z}=0)$ $v(z=l)\;=\bar{v}(\bar{z}=0)$	